Engineering Maths
First Aid Kit

2.27

What is a surd ?

Introduction

In engineering calculations, numbers are often given in **surd form**. This leaflet explains what is meant by surd form, and gives some circumstances in which surd forms arise.

1. Surd form

Suppose we wish to simplify $\sqrt{\frac{1}{4}}$. We can write it as $\frac{1}{2}$. On the other hand, some numbers involving roots, such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt[3]{6}$ cannot be expressed exactly in the form of a fraction. Any number of the form $\sqrt[n]{a}$, which cannot be written as a fraction of two integers is called a **surd**.

Whilst numbers like $\sqrt{2}$ have decimal approximations which can be obtained using a calculator, e.g $\sqrt{2} = 1.414\ldots$, we emphasise that these are **approximations**, whereas the form $\sqrt{2}$ is **exact**.

2. Writing surds in equivalent forms

It is often possible to write surds in equivalent forms. To do this you need to be aware that

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

However, be warned that $\sqrt{a+b}$ is not equal to $\sqrt{a} + \sqrt{b}$.

For example $\sqrt{48}$ can be written

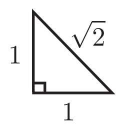
$$\sqrt{3 \times 16} = \sqrt{3} \times \sqrt{16} = 4\sqrt{3}$$

Similarly, $\sqrt{60}$ can be written

$$\sqrt{4 \times 15} = \sqrt{4} \times \sqrt{15} = 2\sqrt{15}$$

3. Applications

Surds arise naturally in a number of applications. For example, by using Pythagoras' theorem we find the length of the hypotenuse of the triangle shown below to be $\sqrt{2}$.



Surds arise in the solution of quadratic equations using the formula. For example the solution of $x^2 + 8x + 1 = 0$ is obtained as

$$\begin{aligned}
x &= \frac{-8 \pm \sqrt{8^2 - 4(1)(1)}}{2} \\
&= \frac{-8 \pm \sqrt{60}}{2} \\
&= \frac{-8 \pm \sqrt{4 \times 15}}{2} \\
&= \frac{-8 \pm 2\sqrt{15}}{2} \\
&= -4 \pm \sqrt{15}
\end{aligned}$$

This answer has been left in surd form.

Exercises

1. Write the following in their simplest forms.

a) $\sqrt{63}$, b) $\sqrt{180}$.

2. By multiplying numerator and denominator by $\sqrt{2} + 1$ show that

$$\frac{1}{\sqrt{2} - 1} \quad \text{is equivalent to} \quad \sqrt{2} + 1$$

The process of rewriting a fraction in this way so that all surds appear in the numerator only, is called **rationalisation**.

3. Rationalise the denominator of a) $\frac{1}{\sqrt{2}}$, b) $\frac{1}{\sqrt{5}}$.

4. Simplify $\sqrt{18} - 2\sqrt{2} + \sqrt{8}$.

Answers

1. a) $3\sqrt{7}$, b) $6\sqrt{5}$. 3. a) $\frac{\sqrt{2}}{2}$, b) $\frac{\sqrt{5}}{5}$. 4. $3\sqrt{2}$.